

Signals & Systems  
 Homework Solutions  
 Problem 56.1

(a)  $m_1 \ddot{d}_1 = k(d_2 - d_1) - c \dot{d}_1$  ,  $m_2 \ddot{d}_2 = f + k(d_1 - d_2)$

(b) States: choose  $x_1 = d_1$  ,  $x_2 = \dot{d}_1$  ,  $x_3 = d_2$  ,  $x_4 = \dot{d}_2$

Then  $\dot{x}_1 = \dot{d}_1 = x_2$  ,  $\dot{x}_2 = \ddot{d}_1 = \frac{k}{m_1} d_2 - \frac{k}{m_1} d_1 - \frac{c}{m_1} \dot{d}_1 = \frac{k}{m_1} x_3 - \frac{k}{m_1} x_1 - \frac{c}{m_1} x_2$

$\dot{x}_3 = \dot{d}_2 = x_4$  ,  $\dot{x}_4 = \ddot{d}_2 = \frac{f}{m_2} + \frac{k}{m_2} d_1 - \frac{k}{m_2} d_2 = \frac{f}{m_2} + \frac{k}{m_2} x_1 - \frac{k}{m_2} x_3$

Inputs:  $u = f$

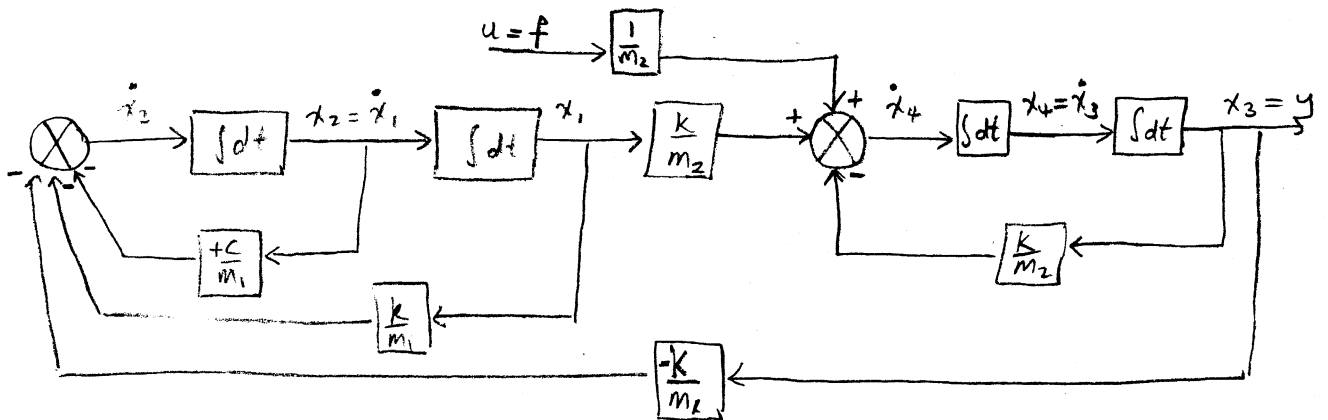
Outputs:  $y = d_2 = x_3$

State-space system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & -\frac{c}{m_1} & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & 0 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix}}_B u$$

$$y = \underbrace{[0 \ 0 \ 1 \ 0]}_C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{[0]}_D u$$

(c)



d) Now  $\vec{y} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} \Rightarrow C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$  ,  $D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$   
 A and B unchanged from (b)

e)  $\lambda_1 = 0$  ,  $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$  ,  $\lambda_{2,3} = -0.02 \pm 1.01j$  ,  $\vec{v}_{2,3} = \begin{bmatrix} -0.029 \pm 0.015j \\ -0.014 \mp 0.029j \\ -0.014 \mp 0.703j \\ 0.71 \end{bmatrix}$  ,  $\lambda_4 = -1.96$  ,  $\vec{v}_4 = \begin{bmatrix} -0.45 \\ 0.87 \\ -0.09 \\ 0.18 \end{bmatrix}$